Mathematical Modeling of the Enhanced Contact Hour (CH+) Metric

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1 Introduction

The mathematical modeling for the enhanced Contact Hour (CH+) metric represents a conceptual shift from a two-dimensional (2D) measurement of regulatory compliance to a three-dimensional (3D) measurement that incorporates program quality in early care and education settings.

2 The Baseline 2D Model (CH)

To calculate the 3D metric, the model first requires the foundational 2D Contact Hour (CH) metric. The CH metric measures the amount of time of interactions between adults and children in a specific space. The base formulas rely on the following variables:

- **TO:** Total time the facility is open (the difference between when the first staff arrives and the last staff leaves).
- **TA:** Number of teaching/caregiving staff.
- **NC:** Number of children on a maximum enrollment day.
- **TH:** Total time children are present (based on when the last child arrives and the first child leaves).

Depending on the specific relationship between how long the children are present (TH) and how long the facility is open (TO), the baseline contact hours can be determined by variations of the following formulas:

$$\mathbf{CH = [NC * (TO + TH) / 2] / TA}$$

$$\mathbf{CH = (NC * TO) / TA}$$

$$\mathbf{CH = [(NC * TO) / 2] / TA}$$

$$\mathbf{CH = NC^2 / TA}$$

These formulas represent a 2D interaction model looking strictly at regulatory compliance based on capacity and time.

3 The Enhanced 3D Model (CH+)

The CH+ metric transforms the 2D 'square' of regulatory compliance into a 3D 'cube' by adding a z-axis representing program quality. This quality is measured using 10 Program

Quality Indicators (PQIs) derived from the *Child Care Early Education Heart Monitor (CCEEHM)(Fiene, 2025)*. The formula for the new enhanced metric is defined as:

$$\mathbf{CH+ = 10 \cdot PQI^3}$$

Key Mathematical Behaviors

- **Exponential Influence:** Because the PQI is cubed, higher ratings across the 10 PQIs exert a significantly greater influence on the final equation. At lower quality values, the formula does not heavily influence the final result.
- **Offsetting Non-Compliance:** The primary function of the CH+ result is to balance or over-compensate for non-compliance in the original CH metric.
- **Application:** This offset is mathematically applied as (CH+) - (CH) when the original CH exceeds acceptable regulatory limits. If the original CH is fully in compliance, the modification has a baseline result of 0 and has no substantive effect on CH+.

4 Integration with the Integrated Regulatory Framework (IRF)

The mathematical modeling of CH+ operates under specific assumptions tied to the *Integrated Regulatory Framework (IRF*)(Fiene, 2026) formula:

$$\mathbf{IRF = (FC) - (F-) - (2F+)}$$

For the CH+ equation to successfully balance out a non-compliant CH result, the model assumes:

- **FC:** There is no non-compliance in licensing key indicators.
- **F-:** Equals 0 (False Negatives).
- **F+:** Equals -2 or less (False Positives).

References

Fiene (2025). *Child Care Early Education Heart Monitor,* Penn State Research Institute for Key Indicators Data Lab and the National Association for Regulatory Administration, unpublished manuscript and app.

Fiene (2026). *Integrated Regulatory Framework Mathematical Model Deciphered and the Interface with the Regulatory Compliance Scale and the Equal Interval and Relative Weighting Risk Assessment Matrix*, Penn State Research Institute for Key Indicators Data Lab, unpublished manuscript.

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